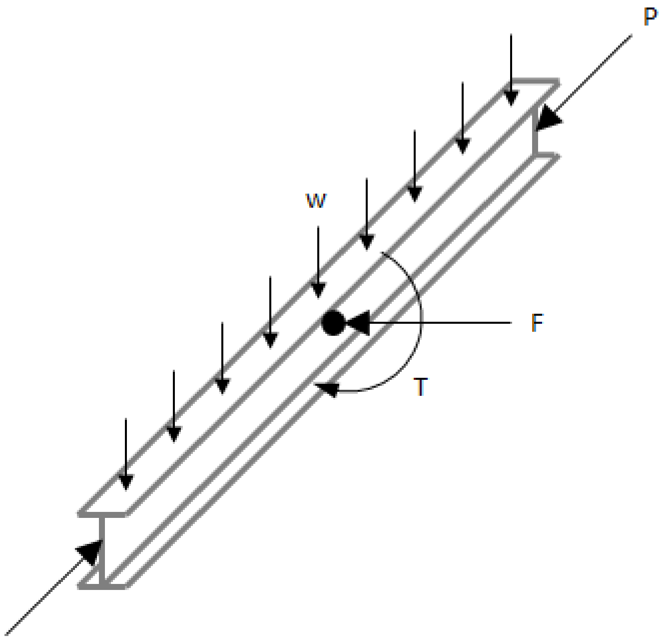
Simply Supported Beam with Torsional and Lateral Loading

This application analyzes a simply supported beam with torsional and lateral loading for a W10X54 steel beam (as defined by the AISC Steel Shapes Database).



References:
- Simplified Design for Torsional Loading of Rolled Steel Members, Lin, P.H., Engineering Journal, AISC, 1977

- 2010 Specification for Structural Steel Buildings (ANSI/AISC 360/10), Fourth Printing (https://www.aisc.org/content.aspx?id=2884)

Parameters

Warping Constant $C_w := 1.2 \times 10^3\ inch^6$

Torsional moment of inertia $J_T := 1.51\ inch^4$

Elastic section modulus about the X-axis $S_x := 60\ inch^3$

Elastic section modulus about the Y-axis $S_y := 20.6\ inch^3$

Cross sectional area of member $A := 15.8\ inch^2$

Plastic section modulus about the x-axis	$Z_x := 66.6\ \mathbf{inch}^3$
Moment of inertia about the x-axis	$I_x := 303\ \mathbf{inch}^4$
Moment of inertia about the y-axis	$I_y := 103\ \mathbf{inch}^4$
Overall depth of member	$d := 10.1\ \mathbf{inch}$
Radius of gyration about the x-axis	$r_x := 4.37\ \mathbf{inch}$

Gravity distributed load	Lateral load in middle	Torsion at mid-span	Axial load
$w := 1.15\ \mathbf{kipf \cdot ft^{-1}}$	$F := 5\ \mathbf{kipf}$	$T := 5.1\ \mathbf{kipf \cdot ft}$	$P := 96\ \mathbf{kipf}$
Beam length	Beam yield stress	Vertical bending unbraced length	Axial vertical unbraced length
$L := 25\ \mathbf{ft}$	$F_y := 50\ \mathbf{ksi}$	$L_b := 15\ \mathbf{ft}$	$L_x := 15\ \mathbf{ft}$
Axial horizontal unbraced length	Young's modulus	Shear modulus	Tortional property (Lin, 1977)
$L_y := 7.5\ \mathbf{ft}$	$E := 29000\ \mathbf{ksi}$	$G := 11200\ \mathbf{ksi}$	$\lambda := \sqrt{\frac{G \cdot J_T}{E \cdot C_w}} = 0.868\ \frac{1}{\mathbf{m}}$

Governing Moments at Middle of Span

Flexural moments

$M_x := w \cdot L^2 / 8 = 89.84\ \mathbf{kipf \cdot foot}$

$M_y := F \cdot L / 4 = 42.37\ \mathbf{kN \cdot m}$

$M_0 := T \cdot L / (4 \cdot d) = 51.35\ \mathbf{kN \cdot m}$

Lin (1977) page 101

$$\beta := \frac{4 \cdot \sinh(\lambda \cdot L/2)^2}{\lambda \cdot L \cdot \sinh(\lambda \cdot L)} = 0.302$$

Torsional moment

$M_T := \beta \cdot M_0 = 11.42\ \mathbf{kipf \cdot foot}$

Check Torsional Capacity (AISC 360-10 H3.3 & Lin, 1977, p100)

Maximum combined normal stress at the load point

$$f_{bx} := \frac{M_x}{S_x} + \frac{2 \cdot M_T}{S_y} = 3.13 \times 10^4\ \frac{\mathbf{lbf}}{\mathbf{in^2}}$$

Safety factor for compression

$\Omega := 1.67$

$F_{nx} := F_y / \Omega = 29.940\ \mathbf{ksi}$

If this is less than 1, then design is satisfactory $f_{bx}/F_{nx} = 1.045$

Check Combined Compression and Bending Capacity (AISC 360-10, H!)

$M_{rx} := \left(M_x/S_x + 2\cdot M_T/S_y\right)\cdot S_x = 156.380\ \mathbf{kipf\cdot foot}$

Effective length factor $K := 0.85$

Elastic buckling stress

$$F_e := \frac{\pi^2\cdot E}{\left(K\cdot L/r_x\right)^2} = 84.06\ \mathbf{ksi}$$

Critical stress

$$F_{cr} := 0.658^{F_y/F_e}\cdot F_y = 38.98\ \mathbf{ksi}$$

Allowable axial strength

$P_n := F_{cr}\cdot A = 2.74\times 10^3\ \mathbf{kN}$

$P_c := P_n/\Omega = 1.64\times 10^3\ \mathbf{kN}$

Available flexural strength (Chapter F AISC 360-10)

$M_n := \min\left(F_y\cdot Z_x, F_y\cdot S_x\right) = 338.95\ \mathbf{kN\cdot m}$

$M_{cx} := M_n/\Omega = 149.70\ \mathbf{kipf\cdot foot}$

This is greater than Mrx so it is satisfactory

$M_{cy} := M_n/\Omega = 202.97\ \mathbf{kN\cdot m}$

This should be below 1 for a satisfactory design

$$\frac{P}{P_c} + \frac{8}{9}\cdot\left(\frac{M_{rx}}{M_{cx}} + \frac{M_y}{M_{cy}}\right) = 1.37$$

Determine Deflections

Max twist angle (Lin 1977, eq4) in degrees

$$\phi := \frac{T}{2\cdot G\cdot J_T\cdot\lambda}\cdot\left(\frac{\lambda\cdot L}{2} - \frac{2\cdot\sinh(\lambda\cdot L/2)}{\sinh(\lambda\cdot L)}\right)\cdot\sinh\left(\frac{\lambda\cdot L}{2}\right) = 3.62$$

$$I_3 := I_x\cdot\cos\left(\frac{(90-\phi)\cdot\pi}{180}\right)^2 + I_y\cdot\sin\left(\frac{(90-\phi)\cdot\pi}{180}\right)^2 = 103.80\ \mathbf{in^4}$$

$$I_4 := I_x\cdot\cos\left(\frac{(90-\phi)\cdot\pi}{180}\right)^2 + I_y\cdot\sin\left(\frac{(90-\phi)\cdot\pi}{180}\right)^2 = 103.80\ \mathbf{in^4}$$

Vertical deflection at middle $\Delta vert := \dfrac{5\cdot w\cdot L^4}{384\cdot E\cdot I_3} = 3.36\ \mathbf{in}$

Horizontal deflection at middle $\Delta horiz := \dfrac{F\cdot L^3}{48\cdot E\cdot I_4} = 0.93\ \mathbf{in}$