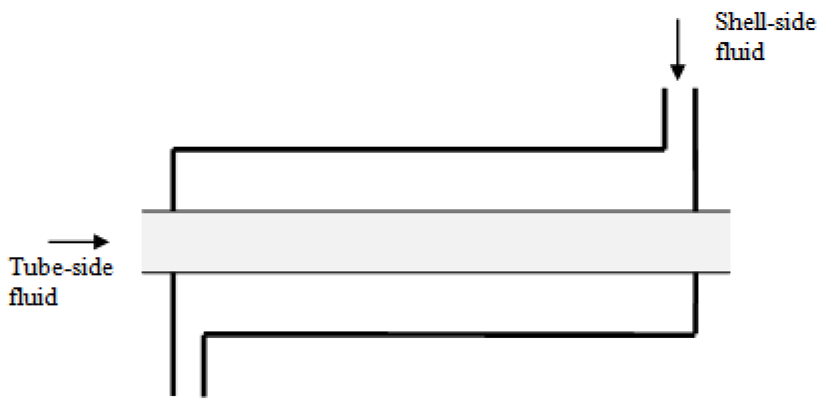
Countercurrent Double-Pipe Heat Exchanger

This application models the temperature dynamics of a counter-current double pipe heat exchanger. Three partial differential equations describe

- heat balances across the tube-side and shell-side liquids
- and a heat balance across the tube wall (taking into account the heat flow from the shell- and tube-side liquids, and conduction along the length of the tube.

The equations are solved numerically, and the temperature profiles are plotted. The heat exchanger is assumed to be perfectly insulated. Densities, specific heat capacities, heat transfer coefficients and thermal conductivities are assumed to be constant.



Length of heat exchanger $L := 1$

Internal and external diameter of inner tube, and internal diameter of outer tube $D_i := 0.05$ $D_o := 0.06$ $D_{is} := 0.1$

	Tube-side fluid	Shell-side fluid	Inner tube wall
Thermal conductivity			$k_w := 106$
Specific heat capacity	$Cp_t := 4085$	$Cp_s := 4186$	$Cp_w := 380$
Density	$\rho_t := 800$	$\rho_s := 900$	$\rho_w := 8000$
Flowrates	$F_t := 1$	$F_s := 1$	
Heat transfer coefficient	$U_t := 40000$	$U_s := 40000$	

Tube wall heat balance

$$pde1 := \frac{\pi}{4} \cdot \left(D_o^{\,2} - D_i^2\right) \cdot Cp_w \cdot \rho_w \cdot \frac{\partial}{\partial t} T_w(x,t) = U_t \cdot \pi \cdot D_i \cdot \left(T_t(x,t) - T_w(x,t)\right) - U_s \cdot \pi \cdot D_o \cdot \left(T_w(x,t) - T_s(x,t)\right) + k_w \cdot \frac{\pi}{4} \cdot \left(D_o^{\,2} - D_i^2\right) \cdot \frac{\partial^2}{\partial x^2} T_w(x,t)$$

Tube-side heat balance

$$pde2 := \rho_t \cdot Cp_t \cdot \frac{\pi}{4} \cdot D_i^2 \cdot \frac{\partial}{\partial t} T_t(x,t) = -Cp_t \cdot F_t \cdot \frac{\partial}{\partial x} T_t(x,t) - \pi \cdot D_i \cdot U_t \cdot \left(T_t(x,t) - T_w(x,t)\right)$$

Shell-side heat balance

$$pde3 := \rho_s \cdot Cp_s \cdot \frac{\pi}{4} \cdot \left(D_{is}^{\,2} - D_o^{\,2}\right) \cdot \frac{\partial}{\partial t} T_s(x,t) = Cp_s \cdot F_s \cdot \frac{\partial}{\partial x} T_s(x,t) + \pi \cdot D_o \cdot U_s \cdot \left(T_w(x,t) - T_s(x,t)\right)$$

Initial and boundary conditions

$$ibc := T_s(x,0) = 300, T_s(L,t) = 300, T_t(x,0) = 360, T_t(0,t) = 360, T_w(x,0) = 330, D[1](T_w)(0,t) = 0, D[1](T_w)(L,t) = 0$$

Numeric solution and plots

sol := pdsolve({pde1, pde2, pde3}, {ibc}, numeric, time = t, range = 0 ..L)

p1 := sol:-plot(T_t, x = 0.5·L, t = 0 ..5, legend = ["Tube-side liquid"], color = black)

p2 := sol:-plot(T_s, x = 0.5·L, t = 0 ..5, legend = ["Shell-side liquid"], color = red)

p3 := sol:-plot(T_w, x = 0.5·L, t = 0 ..5, legend = ["Tube wall"], color = blue)

plots:-display(p1, p2, p3, labels = ["Time (s)", "Temperature (K)"],
title = "Temperature Halfway along Heat Exchanger", labeldirections = [horizontal, vertical],
legendstyle = [location = right]) =