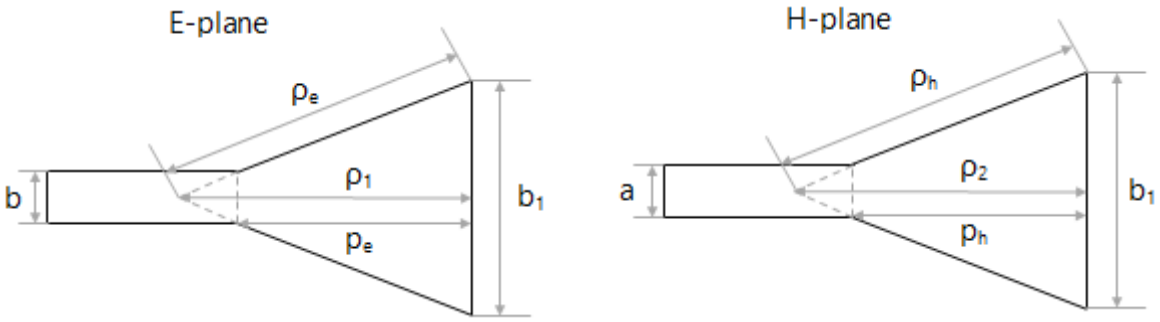
Pyramidal Horn Design

This application calculates the optimum design parameters for an X-band pyramidal antenna.



Parameters

Gain in dB at design frequency	$G_{odb} := 22.6$	$G_o := 10^{G_{odb}/10} = 181.970$
Speed of light	$c := 3 \times 10^{10}\ \mathbf{cm \cdot s^{-1}}$	
Frequency	$f := 11 \cdot 10^{9}\ \mathbf{Hz}$	
Wavelength	$\lambda := \frac{c}{f} = 0.027\ \mathbf{m}$	
Geometrical constraints	$a := 2.286\ \mathbf{cm}$	$b := 1.016\ \mathbf{cm}$

Governing Equation

These are needed for optimum directivity

$$cons1 := G_o = \frac{2 \cdot \pi}{\lambda^2} \cdot a_1 \cdot b_1 \qquad cons2 := a_1 = \sqrt{3 \cdot \lambda \cdot \rho_h} \qquad cons3 := b_1 = \sqrt{2 \cdot \lambda \cdot \rho_e}$$

$$cons4 := p_e = \left(b_1 - b\right) \cdot \sqrt{\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}} \qquad cons5 := p_h = \left(a_1 - a\right) \cdot \sqrt{\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}}$$

$$cons6 := p_e = p_h$$

Numerical Solution

$$\text{res} := \text{fsolve}\left(\{\text{cons1}, \text{cons2}, \text{cons3}, \text{cons4}, \text{cons5}, \text{cons6}\}, \left\{a_1 = 1\,\text{m}, b_1 = 1\,\text{m}, p_e = 1\,\text{m}, p_h = 1\,\text{m}, \rho_e = 1\,\text{m}, \rho_h = 1\,\text{m}\right\}\right)$$

$$\text{res} = \left\{a_1 = 0.166\,\text{m}, b_1 = 0.130\,\text{m}, p_e = 0.280\,\text{m}, p_h = 0.280\,\text{m}, \rho_e = 0.310\,\text{m}, \rho_h = 0.335\,\text{m}\right\}$$

E-Plane Radiation Pattern

$$\rho_1 := \sqrt{\rho_e^{\,2} - \left(\frac{b_1}{2}\right)^2}$$

$$t_1 := \theta \rightarrow \sqrt{\frac{2}{\lambda \cdot \rho_1}} \cdot \left(-\frac{b_1}{2} - \rho_1 \cdot \sin(\theta) \right) \qquad t_2 := \theta \rightarrow \sqrt{\frac{2}{\lambda \cdot \rho_1}} \cdot \left(\frac{b_1}{2} - \rho_1 \cdot \sin(\theta) \right)$$

$$F := \theta \rightarrow \text{FresnelC}\left(t_2(\theta)\right) - \text{FresnelC}\left(t_1(\theta)\right) - (1\,i) \cdot \left(\text{FresnelS}\left(t_2(\theta)\right) - \text{FresnelS}\left(t_1(\theta)\right)\right)$$

$$E_\theta := \theta \rightarrow 20 \cdot \log 10\left(\left|1 + \cos(\theta)\right| \cdot \frac{|F(\theta)|}{|F(0)|} \right)$$

$$\text{plots:-polarplot}\left(\text{eval}\left(E_\theta(\theta), \text{res}\right) + 70,\ \theta = 0..6.28,\ \text{filled} = \text{true}\right) =$$