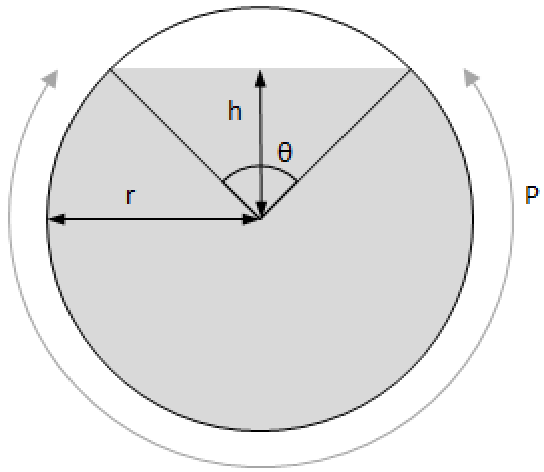
Maximum Flow Rate in Open-Channel Flow for a Circular Pipe

This application determines the greatest attainable flowrate in a circular pipe partially filled with water



The Manning formula is employed to calculate the open-channel flow of water:

$$Q := \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

where
- Q is the flowrate
- n is an empirical coefficient
- A is the cross-sectional area of flow
- R is the hydraulic raduis
- S is the incline of the channel

Flow area for a partially filled circular pipe

$$A := \pi \cdot r^2 - r^2 \cdot \frac{\theta - \sin(\theta)}{2}$$

Wetted perimeter and hydraulic radius

$$P := 2 \cdot \pi \cdot r - r \cdot \theta$$

$$R := \frac{A}{P} = \frac{3.142 \cdot r^2 - r^2 \cdot (0.500 \cdot \theta - 0.500 \cdot \sin(\theta))}{-r \cdot \theta + 6.283 \cdot r}$$

The Manning formula then becomes

$$\text{simplify}(Q) = \frac{0.469 \cdot \left(6.283 - \theta + \sin(\theta)\right) \cdot r^2 \cdot \sqrt{S_0} \cdot \left(\frac{r \cdot \left(-1.000 \cdot \sin(\theta) + \theta - 6.283\right)}{\theta - 6.283}\right)^{2/3}}{n}$$

Parameters

$n := 0.013 \qquad S_0 := 0.0001 \qquad r := 4$

Find the value of theta that maximizes Q

$res := \text{Optimization:-Maximize}(Q) = [98.377, [\theta = 1.005]]$

Maximum flow rate

$Q_{maxflow} := res[1] \qquad \theta_{maxflow} := \text{rhs}(res[2,1]) = 1.005$

Flow depth

$h := r \cdot \cos(0.5 \cdot \theta_{maxflow}) \qquad h + r = 7.505$

$\text{plot}(Q, \theta = 0..1.5) =$