

Radiation Pattern and Directivity of an Antenna Array

This application calculates the array factor and directivity of a uniform linear antenna array, and then plots the radiation pattern.

Number of elements in the uniform array $N := 15$

Design frequency $f_d := 1 \text{ GHz}$

Permittivity and permeability of free space $\epsilon_0 = 8.854 \times 10^{-12} \frac{\text{A}^2 \cdot \text{s}^4}{\text{m}^3 \cdot \text{kg}}$

$$\mu_0 = 1.257 \times 10^{-6} \frac{\text{m} \cdot \text{kg}}{\text{A}^2 \cdot \text{s}^2}$$

Phase constant $\beta_0 := 2 \cdot \pi \cdot f_d \cdot \sqrt{\mu_0 \cdot \epsilon_0} = 20.958 \frac{1}{\text{m}}$

Wavelength $\lambda_d := 2 \cdot \frac{\pi}{\beta_0} = 0.300 \text{ m}$

For a maximum at $\phi_m := \frac{\pi}{3}$

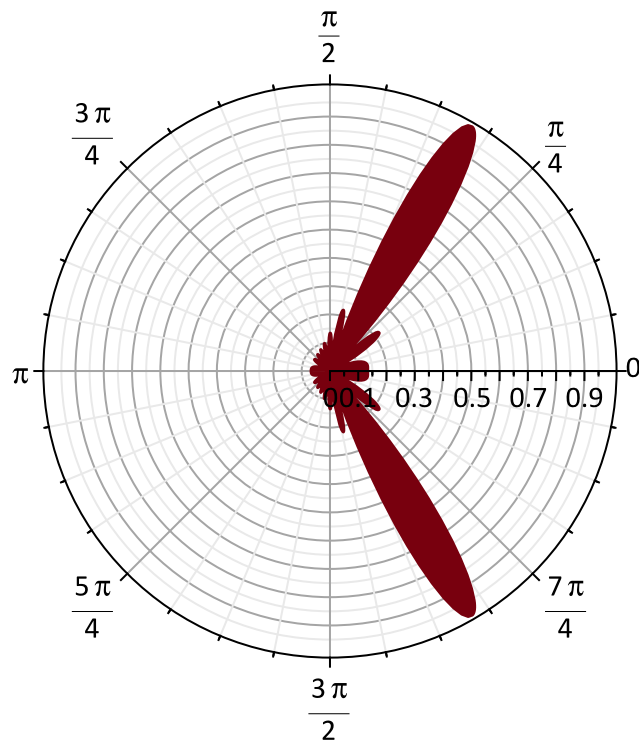
Inter-element spacing $d := \frac{\lambda_d}{3} = 0.100 \text{ m}$

Progressive phase shift between elements $\psi := \beta_0 \cdot d \cdot \cos(\phi_m) = 1.047$

Array factor
$$AF := \left| \frac{1}{N} \cdot \frac{\sin\left(\frac{N}{2} \cdot (\beta_0 \cdot d \cdot \cos(\phi) - \psi)\right)}{\sin\left(\frac{1}{2} \cdot (\beta_0 \cdot d \cdot \cos(\phi) - \psi)\right)} \right|$$

$$AF = 0.067 \cdot \left| \frac{\sin(15.708 \cdot \cos(\phi) - 7.854)}{\sin(1.047 \cdot \cos(\phi) - 0.524)} \right|$$

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plots:-polarplot(AF(φ), φ=0..2·π, filled, transparency=0,
  title="Array Factor", titlefont=[Calibri], axesfont=[Calibri]) =
  Array Factor
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The directivity for this array is calculated from the total power radiated.

$$P_{\text{tot}} := 2 \cdot \int (AF^2, \phi = 0..2 \cdot \pi, \text{numeric}) = 0.942$$

$$D_{\theta} := \frac{4 \cdot \pi}{P_{\text{tot}}} = 13.343$$

which in dB is

$$10 \cdot \log_{10}(D_{\theta}) = 11.252$$