Improving the reliability of multibody simulation by implementing a hysteretic tyre model for radial dynamics

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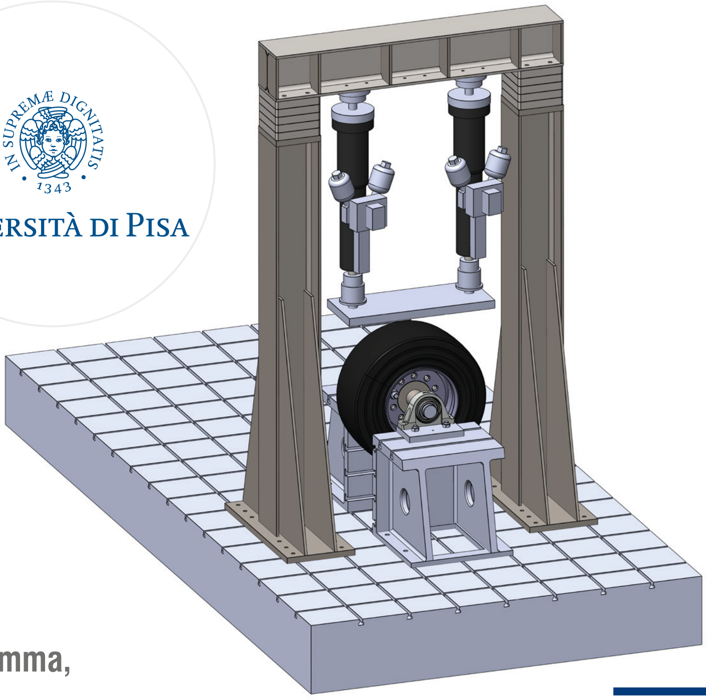

Fig.1. 3D model of the test bench designed to test the tyres (SolidWorks Educational Product. For educational use only).

Multibody simulation is a cornerstone of modern engineering, enabling accurate prediction of structural loads and dynamic behaviour in complex systems. In the automotive and industrial vehicle fields, such simulations are essential for improving design efficiency, durability, and safety. For land vehicles, accurate tyre modelling is critical for reliable simulation results because the forces exchanged between the vehicle and the ground are generated by the tyres.

The vehicle analysed in this study is a forklift from Toyota Material Handling Manufacturing Italia (TMHMI). Forklifts lack a suspension system, so the tyre — characterized by a solid rubber section — plays an even more important role in the vehicle dynamics. The authors' aim is to develop an effective tyre model for radial dynamics to improve the reliability of multibody simulations.

Accurate modelling of tyres has long been a challenge due to their inherently nonlinear and hysteretic behaviour. In this study, a hysteretic Bouc-Wen model was used to capture the radial dynamics of solid rubber tyres. The parameters of the model were obtained from an experimental campaign conducted on the forklift's wheels, ensuring that the model accurately reflects real tyre behaviour.

As a method to validate the performance of the model, the vertical acceleration signals of the front and rear axles of the vehicle during an obstacle crossing (cleat) test were compared with the signals obtained from multibody simulations of the same test, performed using RecurDyn software. The performance of the hysteretic model was then compared with that of a simpler viscoelastic spring damper model to assess the advantages of the hysteretic approach in capturing the complex dynamics of the tyre.

Dynamic wheel behaviour

The dynamic behaviour of the front and rear wheels of the vehicle under study was analysed on a test bench at the University of Pisa (see Fig.1), using static and dynamic tests. The static tests were performed with a triangular displacement profile, while the dynamic tests were carried out with a sinusoidal displacement profile with an amplitude of 0.5mm, varying the preload conditions and frequencies.

The test parameters are given in Table 1.

Test type	Velocity/ Frequency	Preload	Sampling frequency	Number of cycles
Static	0.1mm/s	0mm	20Hz	3 cycles
Dynamic	1/5/10Hz	48/62/76% max displacement	0.1/0.5/1kHz	100 cycles

Table 1. Table of tests carried out.

The results obtained for the front and rear wheels are shown in Fig.2. The figure shows a force-displacement graph illustrating both the static and dynamic cycles. It is clear that the wheels exhibit hysteretic behaviour even when deformed under quasi-static conditions, along with an increase in stiffness during the dynamic tests.

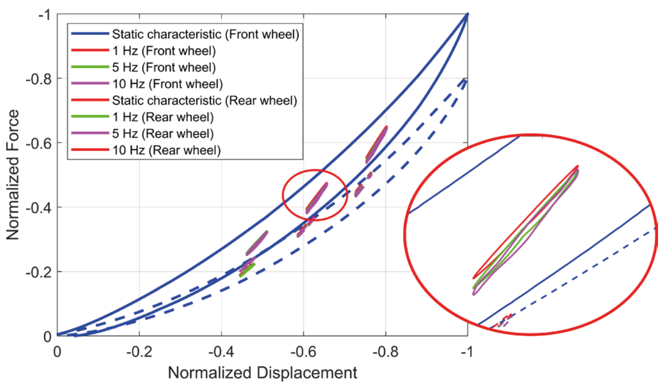

Fig.2: Results of front and rear wheel static and dynamic bench tests and zoom in on dynamic tests with 62% of maximum displacement preloaded.

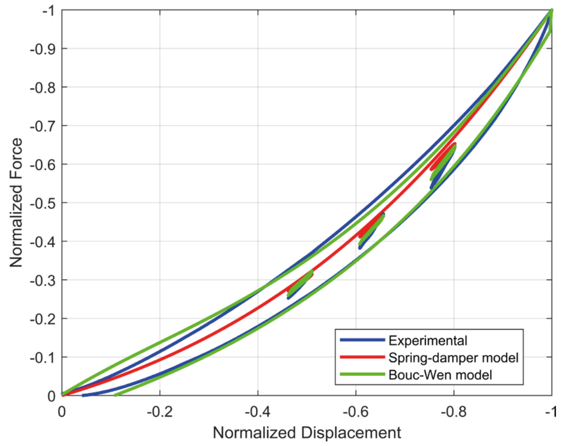

Fig.3. Comparison for the front wheel between models and experiment.

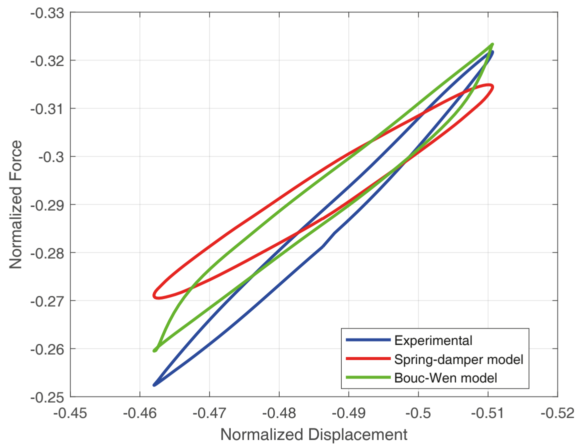

Fig.4. Zoom in on dynamic tests with 48% of maximum displacement preloaded at a frequency of 10Hz.

Wheel	Spring damper model	Bouc-Wen model
Front	6.8%	2.1%
Rear	6.2%	2.4%

Table 2: Relative errors of the fitting models.

Rheological models for wheel dynamics

Two models were proposed and compared to fit the results obtained from the bench tests: a spring damper model and a more complex model consisting of the previous model with an additional force component to evaluate the hysteretic behaviour according to the well-known Bouc-Wen formulation. The model was introduced by Bouc in 1971 [1] and later extended by Wen in 1976 [2].

Since then, the model has been widely used in engineering to describe the nonlinear hysteretic behaviour of materials subjected to cyclic deformation. The equations describing the radial force for the two proposed models is given below. For the spring damper model, the radial force F is expressed as a function of displacement x and the rate of deformation $\dot{x}$ according to the equation.

$$F = k_1x^3 + k_2x^2 + k_3x + c\dot{x}. \quad (eq.1)$$

The model is thus characterized by four parameters: k_1, k_2, k_3 and c. The implementation of the Bouc-Wen model includes the addition of the term αz, where z is obtained by solving a first-order non-linear equation characteristic of the model (Equation 3). The model is therefore characterized by a total of nine parameters.

$$F = k_1x^3 + k_2x^2 + k_3x + c\dot{x} + \alpha z \quad (eq.2)$$

$$\dot{z} = A\dot{x} - \beta|\dot{x}|z|z|^{n-1} - \gamma\dot{x}|z|^{n} \quad (eq.3)$$

The results are shown in Figs.3 and 4. You can see that the spring damper model does not reproduce either the static hysteresis (Fig.3) or the increase in stiffness during the dynamic tests (Fig.4). This results in a higher relative error than the Bouc-Wen model, as shown in Table 2.

Implementation of tyre models in RecurDyn and dynamic simulation of the obstacle crossing test

This section explains how the vehicle, and more specifically the wheels, were modelled to simulate the obstacle crossing test. The multibody model, developed using RecurDyn, is a reduced assembly of the forklift, consisting of the following elements: front axle, chassis, counterweight, rear axle, cab and wheel assemblies. Except for the wheels, all other bodies are rigidly connected by fixed joints.

The wheel assembly, the modelling of which is of particular interest for this study, consists of the rim and the solid tyre. The rims are connected to the axles via planar hinges. The tyres are modelled as rigid bodies.

An in-plane joint limits the displacement in the y-direction (reference system in Fig.5) between tire and rim, while an orientation joint prevents relative rotation between the two bodies. As a result, the wheel system has three degrees of freedom: two associated with the relative displacement between tyre and rim in the x-z plane and one associated with the rotation between rim and axle.

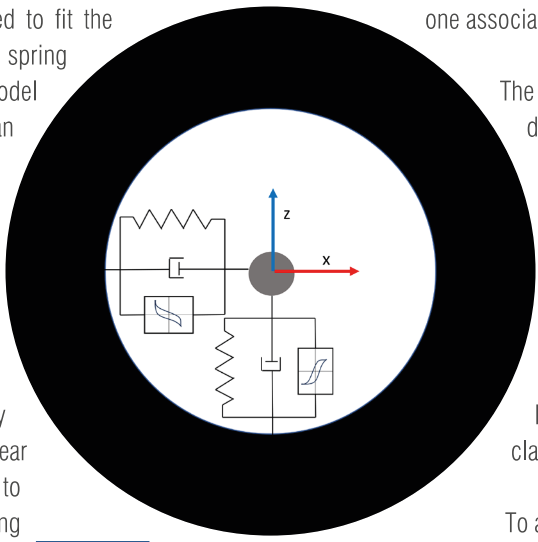

Fig.5. Diagram of the tyre model with force element representation.

The relative displacement in the x and z directions (along with the corresponding velocities) was used as input to calculate the x and z components of the radial force, following the equations of the rheological models presented above. Fig.5 shows a diagram of the wheel with the tyre in black, the axle in grey, and the force elements shown according to the Bouc-Wen model (the rim is omitted for clarity).

To accurately represent the interaction between the wheel and the road, it was necessary to implement contact functions that impose a

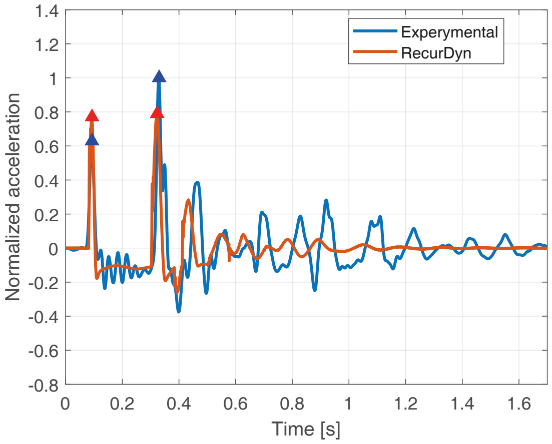

Fig.6. Spring damper model, front accelerometer.

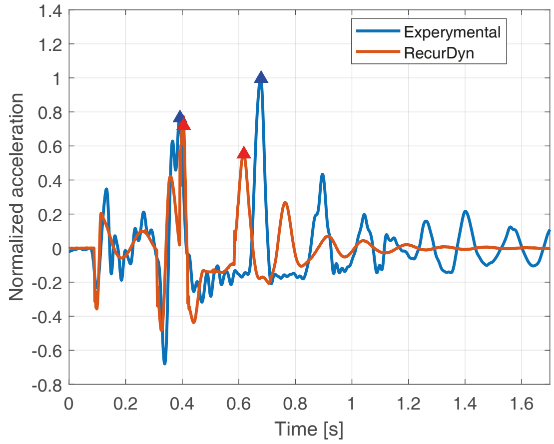

Fig.7. Spring damper model, rear accelerometer.

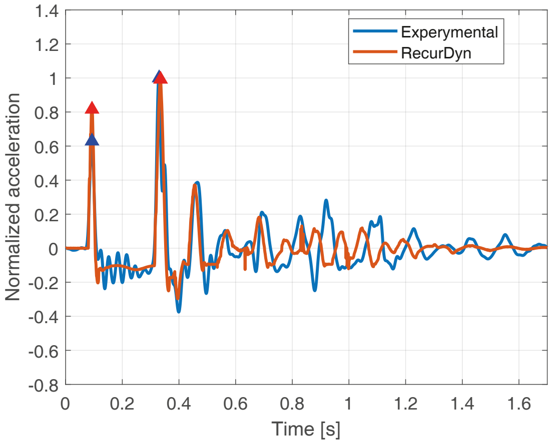

Fig.8. Bouc-Wen model, front accelerometer.

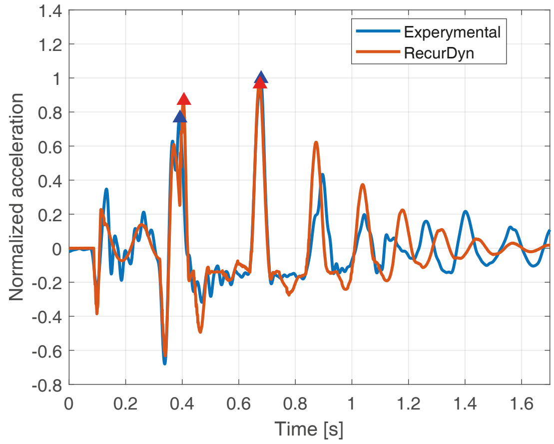

Fig.9. Bouc-Wen model, rear accelerometer.

unilateral constraint, allowing the wheel to lift off the ground, an effect that was observed experimentally after the impact with the obstacle.

The function used is the Geo Curve to Surface contact, where a circle corresponding to the wheel's median plane with a radius equal to that of the wheel is used as the action body, and the ground surface (including the obstacle) is selected as the base body.
The contact parameters were chosen by trial and error, with particular emphasis on the choice of stiffness k. This parameter was set to provide a contact stiffness significantly higher than that of the tyre, so as not to influence the results of the model. In this way, the contact function serves only to impose a unilateral constraint.

The target speed for hitting the obstacle was achieved by applying a velocity profile to the front wheels (driven) that increased to the target angular velocity according to a step function. To achieve this, it was necessary to include friction in the contact function to generate a traction force on the vehicle.

Experimental validation of the models

Experimental validation was carried out by replicating the cleat test with an equipped vehicle. Specifically, two accelerometers were installed on the vehicle: one placed on the front axle and one on the rear axle. The vertical acceleration measured during the experimental campaign was compared with the vertical acceleration signals obtained in RecurDyn from corresponding markers placed in the same positions as the accelerometers.

Figs. 6-9 show the normalized acceleration signals as a function of time. The first column corresponds to the front accelerometer signals, while the second column corresponds to the rear accelerometer signals. The first row compares the experimental signals to those obtained in RecurDyn using the spring damper model. The second row compares the experimental signals with those derived from the Bouc-Wen model.

In the graphs, the peaks corresponding to the impact of the wheel on the obstacle (first peak) and the rebound of the wheel from the ground (second peak) are marked with triangles. The wheel lifting phases can be identified by the time intervals in which the acceleration is approximately constant and negative.

The following observations can be made:

- The spring damper model underestimates the second peak, and the third peak appears earlier. The oscillatory behaviour dampens after a few cycles and there is a reduced number of wheel bounces. This suggests that the system is overdamped.
- In contrast, the Bouc-Wen model accurately predicts both the second and third peaks. This model also shows the correct number of wheel bounces.

These observations suggest that the use of a hysteretic model such as the Bouc-Wen results in an appreciable [3] improvement in the reliability of the simulation compared to a simpler spring damper model.

Conclusion

This study led to the development of two models for the radial dynamics of the solid rubber wheels of a TMHMI forklift: a viscoelastic model (spring damper) and a hysteretic model (Bouc-Wen formulation).

These models were derived from wheel characteristics obtained from bench tests, which showed hysteretic tyre behaviour and an increase in stiffness under frequency-induced deformation.

By fitting these characteristics, the parameters defining each model could be determined. The fitting results show that the spring damper model cannot reproduce the hysteresis observed in static tests and the increase in stiffness observed in dynamic tests, whereas the Bouc-Wen model can, equating to a significantly reduced fitting error.

The model was validated by comparing the experimental vertical accelerations of the front and rear axles with those obtained from multibody dynamic simulations. The comparison shows that the Bouc-Wen hysteretic model provides significantly better simulation accuracy.

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About the Department of Civil and Industrial Engineering of the University of Pisa (DICI)

The Mechanical Design and Machine Construction group of the Department of Civil and Industrial Engineering of the University of Pisa deals with both numerical methods and experimental analyses for the study of complex mechanical structures. Numerical simulations, including FEM and multibody techniques, are often applied to cases from the automotive and other fields, frequently based on input from collaborating companies. Experimental validation is a cornerstone of the group's work and is conducted in the department's state-of-the-art laboratories equipped for static and dynamic characterization of samples, components, and systems. Customized test benches are often developed to address specific industrial challenges.
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