Rubber fatigue ≠ metal fatigue: mean strain effects

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Rubber and metal are very different materials that exhibit vastly different behaviour. Consider the effect of mean strain or stress on the fatigue performance of these materials.

Fig. 1 illustrates a few typical constant amplitude strain cycles, each at a different level of mean strain. If the stress amplitude is equal to the mean stress, we say that we have pulsating tension or fully relaxing tension. If the mean stress is zero, we say that we have fully reversed tension/compression. If the minimum stress is always positive, then we have nonrelaxing tension (i.e. always under load).

Nonrelaxing cycles are quite common in applications. Examples include: pre-loads applied during installation, swaging of a bushing to induce compressive pre-stresses, interference fits, self-stresses occurring due to thermal expansion/contraction, and in tyres, shape-memory effects of textile cords.

In metal fatigue analysis, it is customary to define the effect in terms of stress amplitude σ_a and mean stress σ_m, relative to the yield stress

Type of Constant Amplitude Loading

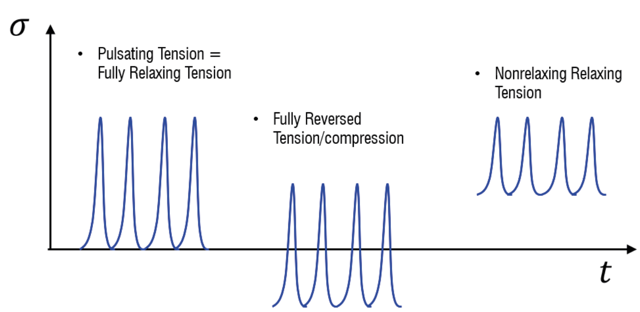


Fig. 1. Constant amplitude cycles at three different mean strains.

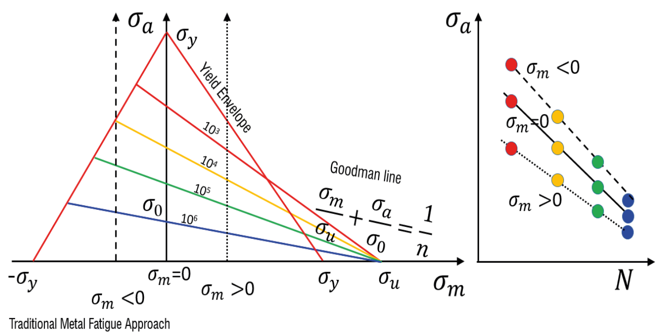


Fig. 2. Haigh diagram (left) and Wohler curves (right) showing mean strain effects on fatigue life for a metal.

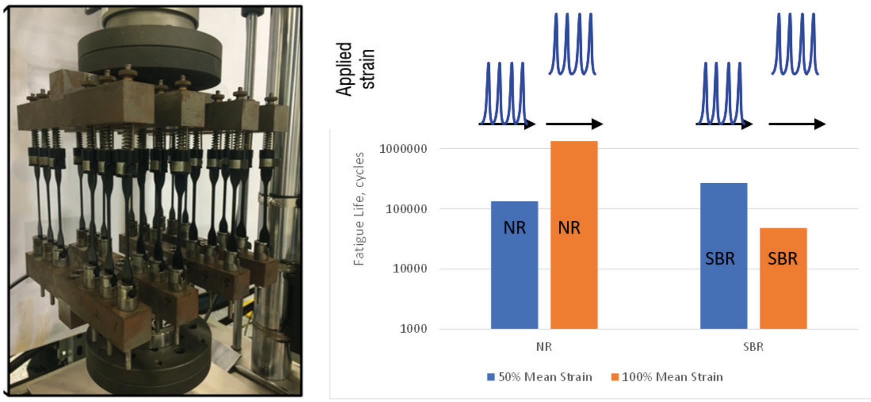
σ_y and the ultimate stress σ_u, as shown in Fig. 2. Below the fatigue threshold stress σ_0, we predict indefinite life. The Haigh (or Goodman) diagram (see the left of Fig. 2) maps fatigue life as a function of these parameters [1]. Wohler curves (see the right of Fig. 2) provide similar information.

For metals, a simple rule may be applied universally: increasing mean strain is detrimental to fatigue life. It is also commonly assumed that the critical plane is perpendicular to maximum principal stress direction.

There are many ways that rubber materials differ from metallic ones:

- At the atomic scale, rubber is composed of long chain molecules experiencing constant thermal motion while interlinked with a permanent network topology. This structure permits large, elastic/reversible straining to occur. Metals could not be more different, existing as individual atoms packed into well-ordered crystals with occasional dislocations or lattice vacancies. This structure permits only vanishingly small strains before inelastic deformation occurs.

Mean Strain Effect in Pulsating Tension: NR vs. SBR



Ramachandran, Anantharaman, Ross P. Wietham, Sunil I. Mathew, W. V. Mars, and M. A. Bauman. "Critical plane selection under nonrelaxing simple tension with strain crystallization." In *Fall 192nd technical meeting of the rubber division*, pp. 10-12. 2017.

Fig. 3. Fatigue tests run in simple tension under constant amplitude show a significant increase in life for natural rubber (NR), which strain crystallizes, and a decrease of life for styrene butadiene rubber (SBR) which is amorphous [2].

	Amorphous Rubber	Strain Crystallizing Rubber
Crack Growth Rate Law	$r = r_c \left(\frac{T_{eq}}{T_c}\right)^F$	
Equivalent Fully Relaxing Tearing Energy	$T_{eq} = \Delta T$ $T_{eq} = T_{max} - T_{min}$ $T_{eq} = T_{max}(1 - R)$	$T_{eq} = T_{\max,R}^{\frac{F(R)}{F(0)}} T_c^{\left(1-\frac{F(R)}{F(0)}\right)}$
Crystallization effect	None	$F(R) = F_0 e^{F_{exp} R}$

Table 1. Models for computing crack growth rate in amorphous and strain-crystallizing rubbers.

- At the meso scale, rubber is typically a composite material containing fillers such as carbon black, silica, or clay, as well as other chemical agents. The mesoscale of a metal is generally described in terms of crystalline grain boundaries and inclusions or voids. Rubber exhibits many "special effects" that are not seen in metals: rate and temperature dependence, ageing, cyclic softening.

It is unsurprising therefore that analysis methods for rubber differ substantially from those applied for metals. Rubber's fatigue performance has a more complex dependence on mean strain. For amorphous (i.e. non-crystallizing) rubbers, increasing mean strain reduces the fatigue life, as with metals. But for rubbers that exhibit strain-induced crystallization, mean strain can greatly increase fatigue life, as illustrated in Fig. 3. Fatigue simulations therefore must take account of the strain crystallization effect.

Mean strain effects are specified in the Endurica fatigue code in terms of fracture mechanical behaviour, using the concept of an equivalent fully relaxing tearing energy T_{eq}. The tearing energy for fully relaxing conditions is considered equivalent when it produces the same rate of crack growth as the nonrelaxing condition. For amorphous rubbers, the equivalent $R=0$ tearing energy T_{eq} is simply the range ΔT of the tearing energy cycle, which can be expressed in terms of the min. and max. tearing energies T_{min} and T_{max}, or in terms of $R=T_{min}/T_{max}$.

Plugging this rule into the power law crack growth rate function yields the well-known Paris law, which predicts faster crack growth for increasing mean strain. For a strain crystallizing rubber, the equivalent fully relaxing tearing energy can be specified using the Mars-Fatemi law. In this case, the equivalent fully relaxing tearing energy depends on a function F(R), which specifies the crystallization effect in terms of its influence on the power law slope of the crack growth rate law. The relationship for amorphous and crystallizing rubbers is summarized in Table 1 [3,4].

The fatigue behaviour of rubber can be charted in a Haigh diagram, but the contours can be quite different from those of metal. In metal fatigue analysis, we assume that cracks always develop perpendicular to the max. principal stress direction. This is not always true for rubber, especially in cases involving strain crystallization and nonrelaxing loads.

For rubber fatigue analysis one must therefore use critical plane analysis [5], in which fatigue life is computed for many potential crack orientations, and in which the crack plane with the shortest life is identified as the most critical plane. Fig. 4 shows the dependence of the fatigue life and the critical plane orientation on strain amplitude and mean strain. A sphere is plotted for each pair of strain amplitude and mean strain coordinates on which the colours represent fatigue life, and unit normal vectors indicate critical plane orientations. Different combinations of mean strain and strain amplitude can produce a range of crack plane orientations.

The Haigh diagrams for NR and for SBR are shown in Fig. 5. In these images, red represents short fatigue life and blue long life. For NR (on the left), the long-life region of the Haigh diagram exhibits a notable dome-like shape, indicative of a beneficial effect of mean strain under the influence of strain-induced

Critical Plane Analysis of Haigh Diagram

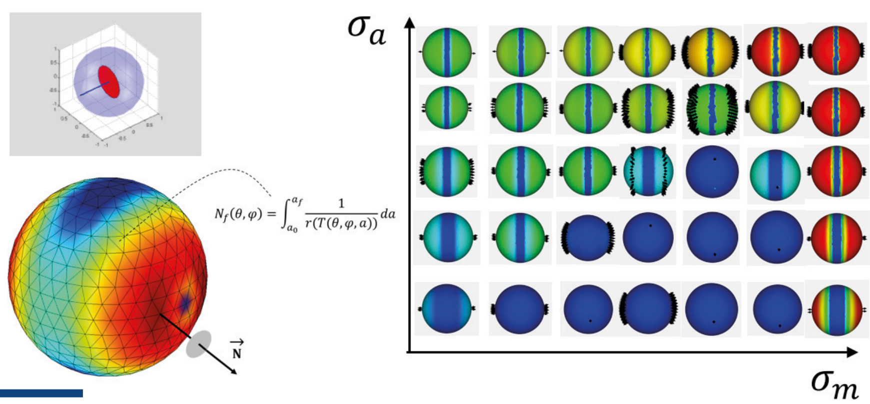


Fig. 4. Critical plane analysis consists of integrating the crack growth rate law for every possible crack orientation, and identifying the orientation that produces the shortest life (left). Each point in the Haigh diagram (right) is associated with its own critical plane orientation.

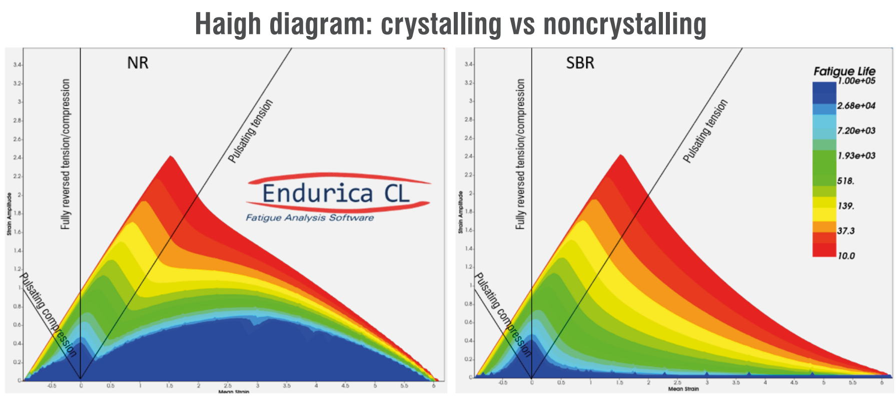


Fig. 5. Haigh diagrams computed for NR (left) and for SBR (right) rubbers.

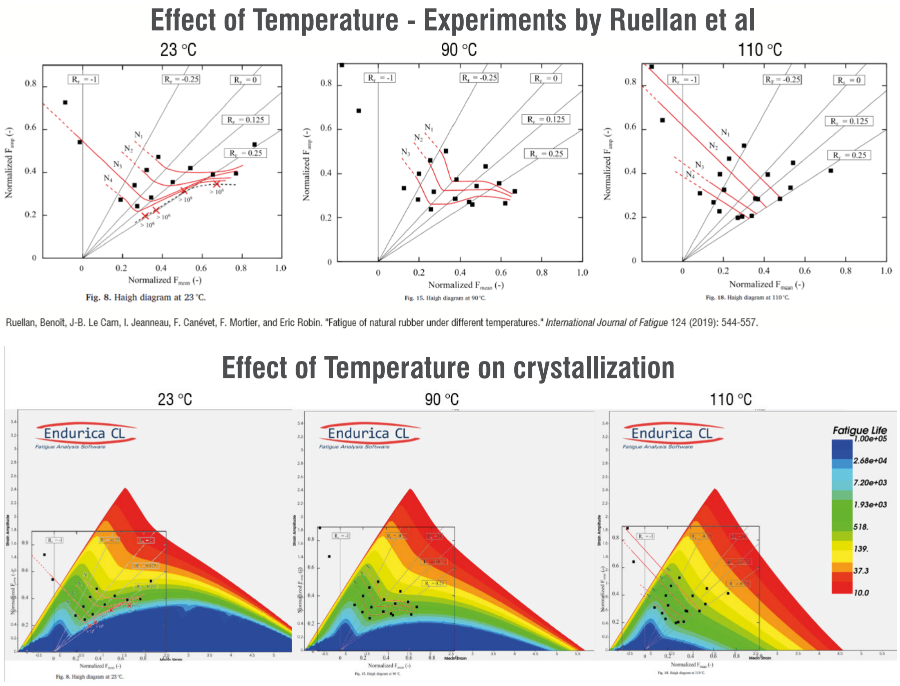


Fig. 6. Experimental Haigh diagram [6] for NR at three temperatures (top), compared to computed Haigh diagram (bottom). Increasing temperature tends to reduce the beneficial effect of strain crystallization.

crystallization. In contrast, SBR always exhibits decreased fatigue life as mean strain increases. Even so, the Haigh diagram for SBR has a nonlinear character associated with the material's hyper elasticity that is also distinct from a metal. It should be noted that the strain crystallization effect in rubber depends on temperature. At colder temperatures the effect is stronger and at higher temperatures it is weaker.

Fig. 6 compares experimental Haigh diagrams [6] (top) for a crystallizing rubber to computed results (bottom) for three temperatures.

In summary, while tensile mean stresses are always detrimental in metals, in rubber they may be either beneficial or harmful depending on whether the rubber can strain crystallize. The benefits of mean stresses in rubber can be quite strong - sometimes amounting to more than several orders of magnitude. The beneficial effect is stronger at colder temperatures and is reduced at higher temperatures.

Critical plane analysis is essential for accurately predicting the effects of strain crystallization in rubber. Wohler curves, commonly used for metal fatigue analysis, incorrectly assume that the worst-case plane is always normal to the max. principal stress direction. This is not an accurate approach for strain crystallizing rubber under mean strain. Use the Endurica fatigue solvers to accurately capture these effects when it is important to get durability right!

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